Problem Set 3: Radio Telescopes and Stellar Remnants [Answer Key]

Answer each question in 1 or 2 complete sentences, showing your work (math equations or illustrations) as needed.

1. The Five-hundred metre Aperture Spherical Telescope (FAST) is the largest radio telescope in the world, with a 500 meter dish. It observes from 0.07 - 3 Gigahertz. But it cost $180 million dollars! Let's see if we can make an interferometer that's cheaper and just as sensitive.
    a. If we build our interferometer using 5-meter radio antennas, each individual antenna is 10000 times less sensitive than FAST. How many 5-meter telescopes will we need to match the sensitivity of FAST? *(Hint: the radiometer sensitivity for an interferometer is* $\sigma \approx \frac{T_{sys}/G}{N\sqrt{\Delta\nu t_{int}}}$*, where N is the number of antennas, Tsys is the system temperature, G is the gain,* $\Delta\nu$ *is the bandwidth, and* t_{int} *is the integration time.)*

First, we know the sensitivity will be 10000 times smaller for the 5 meter antennas than for the 500 meter FAST telescope. That means that T_{sys}/G is 10000 times smaller. They'll have the same bandwidth and integration time, so we can find the number of antennas using:

$$\sigma \approx \frac{T_{sys}/G}{N\sqrt{\Delta\nu t_{int}}} = \frac{\sqrt{2}\sigma_{FAST}}{N}$$

Multiplying both sides by N, we can plug in the 10000:1 ratio for sensitivities:

$$N = \frac{\sqrt{2}\sigma_{FAST}}{\sigma} = 14142 \sim 15000$$

So we would need 15000 antennas! (Its ok if you miss the factor $\sqrt{2}$)

    b. If each antenna is $180k, we'll only have enough for 1000 antennas. How much should we increase the integration time to match FAST's sensitivity with 1000 antennas?

With only 1000 antennas, we need to increase the observing time to match FAST's sensitivity. Setting the single telescope equal to the interferometer sensitivity:

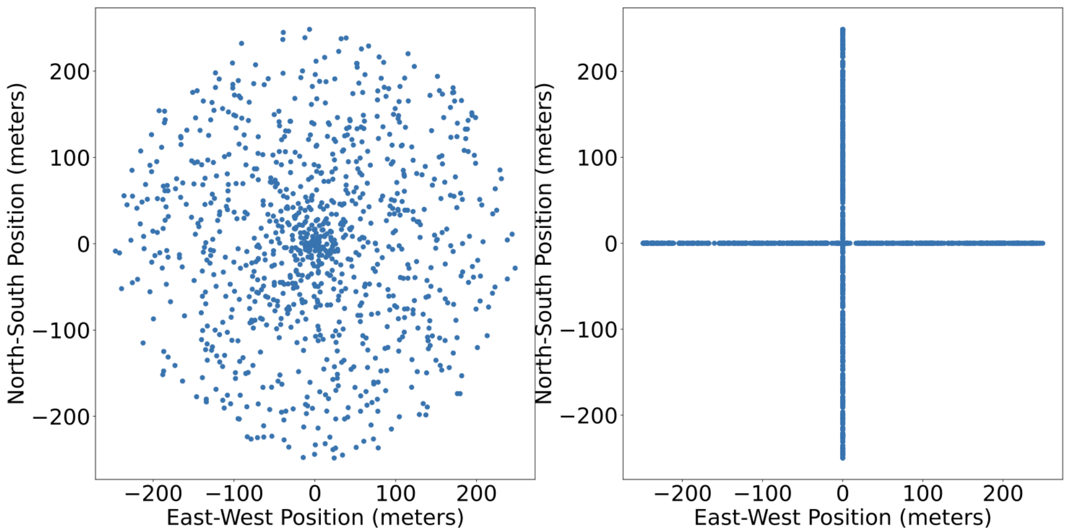
$\sigma = \sigma_{FAST}$

$\frac{10000}{N\sqrt{t_{int}}} = \frac{1}{\sqrt{2t_{int,FAST}}}$

$\frac{t_{int}}{t_{int,FAST}} = 2(10000/N)^2 = 2(10000/1000)^2 = 200$

So we must use 200 $\times$ longer integration times.

c. Below are two options for how to arrange our antennas; which should we use and why? (*Hint: think about the fringe patterns)*



We want to use the left option, where antennas are randomly scattered. This is so that the fringe patterns for each pair of antennas will be oriented in different directions. When they superimpose and interfere, we'll suppress the fringe pattern, and have less prominent sidelobes. For the right option, the fringes for pairs of antennas within the same arm will superimpose and make sidelobes more significant.

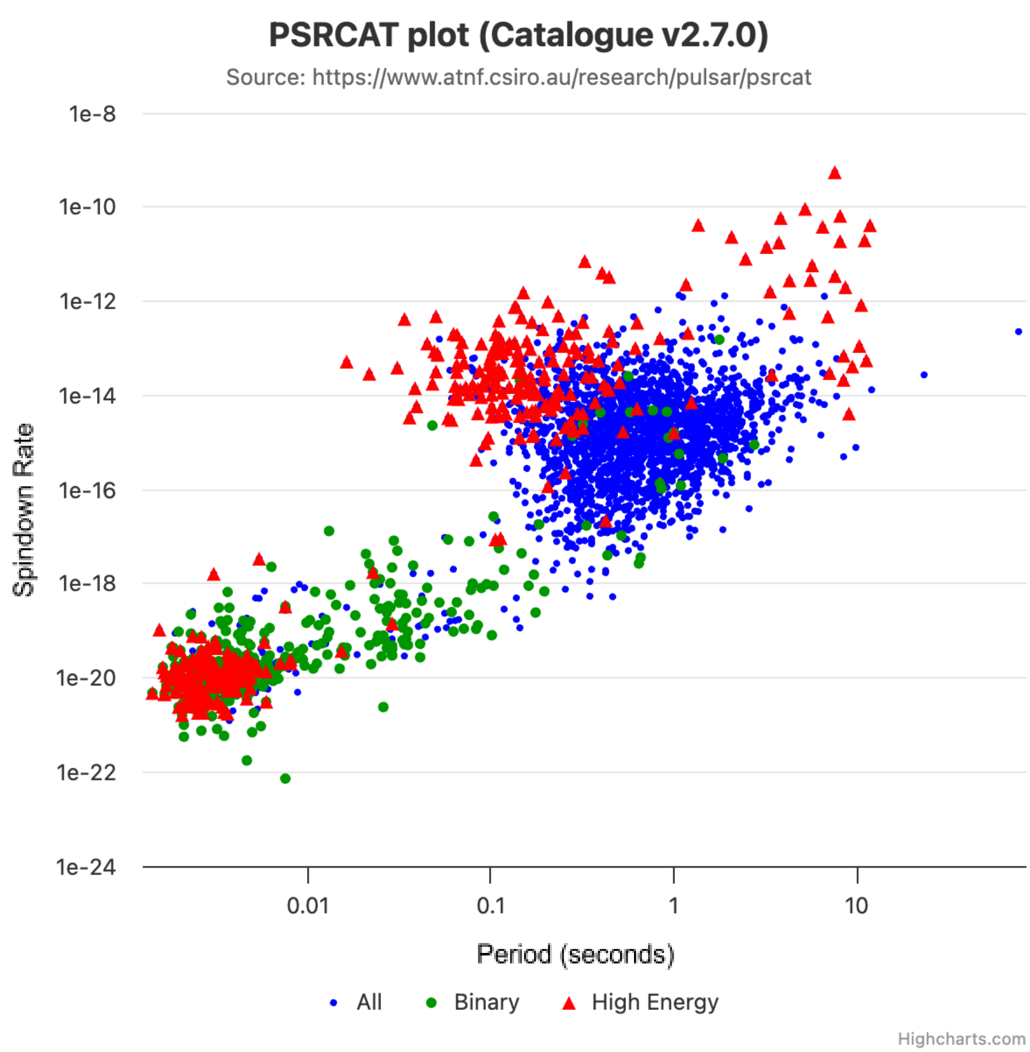
d. ***Bonus:*** What is the full-width at half-max (FWHM) of FAST's primary beam? What about for each small telescope? What is the FWHM of the interferometer's synthesized beam if the antennas are randomly spread over a 500m circle (left figure)? Why is this an advantage for the interferometer? (*Hint: the full-width half max is* $\theta \approx 1.22\lambda/D$*, where you can use* $\lambda = c/1.4$ *GHz and D is the effective diameter)*

First, we convert the frequency to a wavelength:

$\lambda = c/\nu = \frac{3\times10^8 m/s}{1.4\times10^9 Hz} = 21cm$

Then, plugging this in to the FWHM equation:
$\theta = 1.22\lambda/D = 1.22 \times (0.021m)/500m = 0.00005\, radians$
Similarly, for the small telescopes:
$\theta = 1.22\lambda/D = 1.22 \times (0.021m)/5m = 0.005\, radians$
For the synthesized beam, remember the effective diameter is the maximum baseline length which is 500 meters. So the synthesized beam has the same FWHM as FAST, 0.00005 radians.
This is a benefit for the interferometer because each antenna is sensitive to a larger region of the sky, but smaller synthesized beams can still be formed within the field of view to achieve the same spatial resolution as FAST.

2. We've found a new pulsar, but it looks a bit…off. You were using the Green Bank telescope to search for pulsars and see radio pulses that repeat every 18.18 minutes! After monitoring the pulsar for a while, we find it has a spin-down rate of $\dot{P} = 6 \times 10^{-10} s/s$.
    a. What is different about this pulsar compared to other pulsars? For reference, here's a plot of the period (x-axis, shown in seconds) and spin-down rate (y-axis) of pulsars from the ATNF Pulsar Catalog:



18.18 minutes is much longer than the other pulsars shown on the plot. Converting to seconds, the period is 1090.8 seconds; that's about 10x longer than the longest pulsar period shown.

b. If we assume this is a pulsar which started with a rotation period close to 0, approximately how long (in years) has it been a pulsar? (*Hint: We can estimate a pulsar's age as* $\tau_C \approx P/2\dot{P}$)

Pulsars expend their rotational energy through their emission, so their rotation periods get longer over time. If we assume it starts with a very small period, we can estimate the age using the rate of change of the pulse period:

$$\tau_c \approx \frac{P}{\dot{P}} = \frac{1090.8\ s}{2(6\times10^{-10})} = 900\ billion\ seconds$$

Converting this to years:

$$(900 \times 10^{9} s) \times \frac{1\ day}{86400\ s} \times \frac{1\ yr}{365\ days} = 28\ thousand\ years$$

c. The brightest pulses have a radio luminosity $L = 4 \times 10^{31}$ erg/s. Most pulsars use their rotational kinetic energy to power their radio emission. How long (in years) could this 18.18 minute pulsar keep producing $L = 4 \times 10^{31}$ erg/s pulses? Compare this to the age: could it have been radiating for its full age? *(Hint: the rotational energy in ergs is* $E_{rot} \approx \frac{4\pi^2\times10^{45}}{2P^2}$*)*

First, let's find the total rotational energy by plugging in the pulse period:

$$E_{rot} = \frac{4\pi^2\times10^{45}}{2(1090.8s)^2} = 1.6 \times 10^{40} erg$$

Then, we divide by the luminosity, which is the energy radiated per unit time:

$$\frac{E_{rot}}{L} = \frac{1.6\times10^{40} erg}{4\times10^{31} erg/s} = 414\ billion\ seconds$$

Converting to years:

$$414\ billion\ seconds \times \frac{1\ day}{86400\ s} \times \frac{1\ yr}{365\ days} = 13.15\ years$$

This is much shorter than the characteristic age, so the pulsar would have run out of energy very quickly and could not sustain its luminosity for its full age.

d. Another theory is that this is not a pulsar, but a White Dwarf in binary orbit with an M-dwarf star, which usually has an intrinsic optical magnitude $M_V \approx 10$. If the distance is 1400 parsecs, and the extinction is around $A_V \approx 10$, what would the apparent optical magnitude be? (*Hint:* $m_V = M_V + A_V + 5log_{10}(d/1pc) - 5$*, and* $log_{10}(1400) = 3.14$*)*

Plugging into the equation, we can get the apparent magnitude:

$m_V = M_V + A_V + 5log_{10}(d/1pc) - 5$

$m_V = 10 + 10 + 5log_{10}(1400) - 5 = 30.7$

This is waaaay to faint for even the most sensitive space-based optical telescopes to detect, which usually can detect to around 20-25 mag.

This is problem is based on a real source called GLEAM-X J162759.5-523504.3, which you can read about in this article by Natasha Hurley-Walker et al. https://www.nature.com/articles/s41586-021-04272-x . There have been around 14 of these 'Long Period Radio Transients' discovered in the past 4 years; two of them are White Dwarf-M Dwarf binaries, but so far, we don't know what the others are!