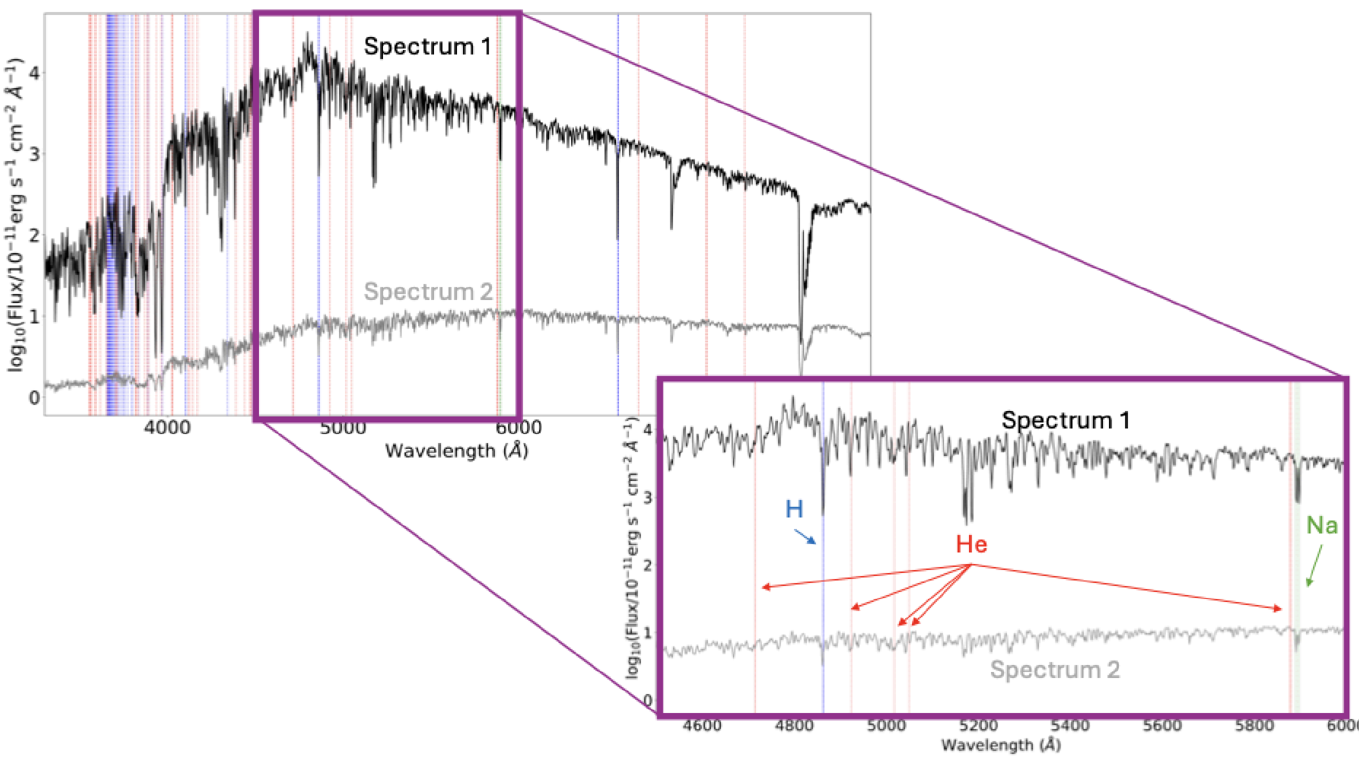
Problem Set 2: Stellar Spectra and Refraction [Answer Key]

Answer each question in 1 or 2 complete sentences, showing your work (math equations or illustrations) as needed.

1. The figure below shows the optical spectra of two stars (black and grey). The region outlined in purple shows a smaller wavelength range from 4500 to 6000 Angstroms (an Angstrom, Å, is 0.0000000001 meters). The blue lines are spectral lines of Hydrogen (H), the red lines are spectral lines of Helium (He), and the green lines are a set of spectral lines of Sodium (Na) called the "Sodium Doublet".



   a. Based on the shape of the spectra, which star (1 or 2) do you think is *hotter* and why? (*Hint: Remember that stars have blackbody spectra, so they follow Wein's law,* $\lambda_{peak} T = constant$)

We can see that while Spectrum 2 is flatter than Spectrum 1, it still has a peak. The peak wavelength of Spectrum 1 is around 4800 Å while the peak wavelength of Spectrum 2 is around 6000 Å. From Wein's law, we see that the peak wavelength is *inversely proportional* to the temperature; a larger peak wavelength means a lower temperature to keep $\lambda_{peak}T = constant$. Therefore, since Star 1 has a smaller λ_{peak}, it is hotter than Star 2.

b. One of these is a main sequence star, while the other is a supergiant star. Which is which, and why?

After a main sequence star finishes burning its Hydrogen, the core heats up to a point when it can burn Helium. *But* the envelope of the star will start to expand and thus will cool down as it becomes a giant star. The temperature that determines the blackbody spectra is the temperature of the envelope, so the colder star is the supergiant, while the hotter star is the main sequence star. Based on our answer to question a, this means Star 1 is the Main Sequence star and Star 2 is the Supergiant star.

c. Recall that a supergiant star has already burned Hydrogen and Helium in its core, but will have an envelope of light elements including Hydrogen and Helium. But we don't see any Helium lines in either star. Why do you think this is?

To observe absorption lines, remember that electrons have to be excited to higher energy levels by photons. Since we don't see Helium lines, this means the star isn't hot enough to create energetic photons needed to excite Helium electrons to high energy levels.

d. ***Bonus:*** Both stars have the Sodium Doublet absorption line; why might a Main Sequence star, which has not fused any Sodium in its core yet, have Sodium in its envelope?

The only way for this to happen is if the gas that collapsed to form the star already contained Sodium. As we'll learn about in lecture 4, these must be Population I stars, which formed after the first stars (Population II) exploded and released heavy metals into the interstellar medium.

2. Let's see why stars twinkle and planets don't! As light passes through the atmosphere, the intensity can fluctuate due to variations in the density which change the index of refraction of the air. Let's model the atmosphere as a screen of lenses with index of refraction $n = 1.00003$ at a height of 1000 meters. If you'd like to learn more, this question is based on an article, "Why do stars twinkle, and do they twinkle on Mars?" by Lorne Whitehead, John Huizinga, and Michele Mossman, which you can find here: https://pubs.aip.org/aapt/ajp/article/80/11/980/1056885/Why-do-stars-twinkle-and-do-they-twinkle-on-Mars
    a. When a star twinkles, it doesn't seem to change position; so when its light passes through the lens of the atmosphere, it's deflected by less than 0.0002 radians (0.01 degrees), the angular resolution of our eyes. What must be the radius of curvature of the atmospheric lenses? (*Hint: radius of curvature is approximately* $r_c \approx f\theta$, *where f is the focal length and* θ *is the angle of deflection)*

Since we are observing from the ground, we want the light to focus at our eyes. So the focal length f is the atmospheric height 1000 meters. Multiplying this by the angular resolution 0.0002 radians gives the radius of curvature:

$r_c \approx f\theta = 1000\, meters \times 0.0002\, radians = 0.2\, meters$

So the lenses must have 20 centimeter radii of curvature.

    b. Stars twinkle roughly every 0.1 seconds. If the atmosphere fluctuations move at a speed of around 1 meters per second, what's the diameter of each lens? (The size scale of fluctuations)?

Remember: $distance = speed \times time$. We can find the diameter of each lens by multiplying their speed by the timespan we observe the stars' twinkle on:

$D = 1\, m/s \times 0.1\, s = 0.1\, meters$

So each lens is roughly 10 centimeters in diameter.

    c. A sun-like star has a diameter of 1.4 billion meters (1,400,000,000 meters); if a star's angular size on the sky must be smaller than 0.0002 radians (0.01 degrees) for us to see it twinkle, how far away (in Astronomical Units, AU) must it be? (*Hint: a star of diameter d at distance l will have an angular size* $\theta \approx d/l$; $1\, AU = 1.5 \times 10^{11}$ *meters*)

For a star with diameter $d = 1.4\ billion\ meters$ to appear at angular size $\theta = 0.0002$ radians, we need to find the distance:
$l = d/\theta = 1.4e9\ meters/0.0002 radians = 7\ trillion\ meters$
Converting this to AU:
$\frac{7\times10^{12} m}{1.5\times10^{11} m/AU} = 46\ AU$
For reference, that's almost twice as far as Neptune (30 AU).

d. Mars is one of the brightest planets we can see from Earth; it has a diameter of 6.780 billion meters and is 228 billion meters away. Does Mars twinkle? Why or why not?

Let's use the same equation as the last part, but rearrange it:
$\theta = d/l = 6.78e9\ m\ /\ 228e9\ m = 0.03\ radians$
Since 0.03 radians > 0.0002 radians, Mars is too large/nearby to see it twinkle. In fact, this is also the case for other planets we can see with the naked eye (Venus, Jupiter, Mercury, Saturn).