

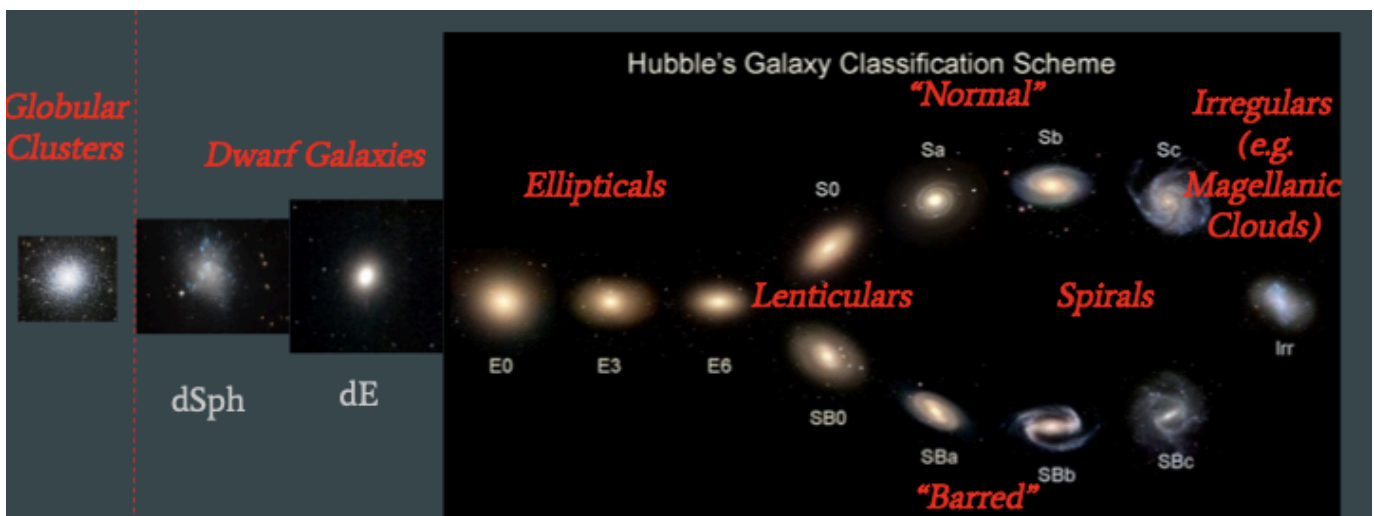
Lecture 4: Large-Scale Structure and Cosmology

[Answer Key]

Galaxies and Dark Matter

A **Galaxy** is a gravitationally-bound collection of stars, gas, and dust, like our galaxy, the Milky Way

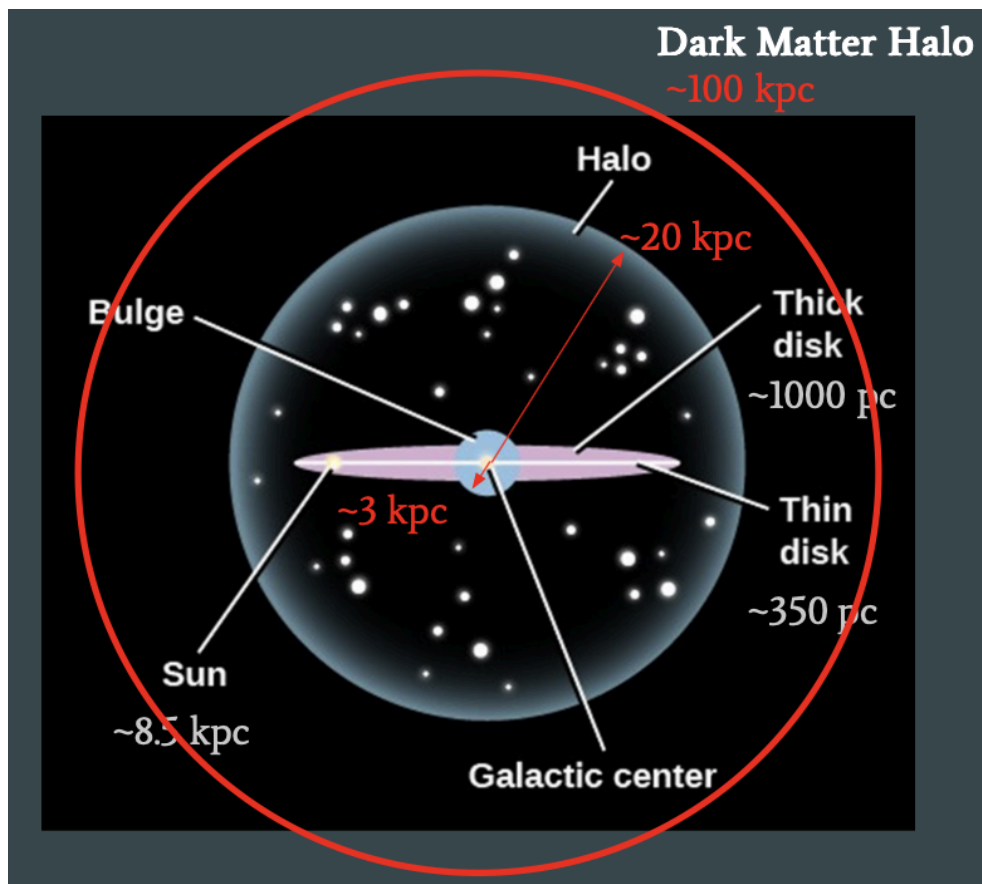
- Galaxy masses range from $10^7 - 10^{13} M_{\odot}$, with diameters from 1 – 100 kiloparsecs
- **Hubble Sequence** - morphological galaxy classifications
 - “Early type”: Dwarf, Elliptical, Lenticular galaxies [**NOT actually younger or older**]
 - “Late type”: Spiral, Irregular galaxies [**NOT actually younger or older**]
 - Subclasses are based on the **brightness** of bulge and disk, the **tightness** of **spiral arms**, and how well stars and HII emission nebulae are **resolved** in spiral arms



- **Galaxy Group** - gravitationally bound collection of 50 or less galaxies
 - The **Local Group** consists of the **Milky Way**, The Andromeda Galaxy (M31), and many orbiting **satellite** galaxies
- **Galaxy Cluster** - gravitationally bound collection of hundreds to thousands of galaxies
 - **Field galaxies** - galaxies not apparently bound to galaxy clusters

- **Galaxy Supercluster** - largest known gravitationally bound structure in the universe, containing sub-clusters and superclusters
 - The **Local Group** is part of the **Virgo** supercluster, which is part of the **Laniakea Supercluster**
- **Luminosity function** - distribution of galaxy luminosities
 - Closely follows **Schechter function** for early (elliptical) galaxies
 - Deviates from **Schechter function** for late (spiral, irregular) galaxies; fewer low luminosity galaxies than expected

Structure of the Milky Way



- **Population I stars:**
 - **Young, metal-rich** stars formed from heavy elements ejected from the supernovae explosions of the first stars in the galaxy
 - Mostly found in **thin disk**, but can also be found in the bulge
- **Population II stars:**
 - **Old, metal poor** stars formed from primordial gas before the first stars exploded
 - Found in the **bulge** and **halo**

- **Population III stars** haven't been confirmed, but are theorized to form from primordial H and He shortly after Big Bang
- **Star Formation History** - star formation rate as a function of Galaxy age; remains complex, population model insufficient
- **Dark Matter** - non-luminous matter that makes up ~90% of the universe, found in the **dark matter halo** of galaxies based on the gravitational effects
- Galaxy formation is still being explored, but most evidence suggests a **hierarchical** formation process:
 - [1] **The Cosmic Web** - Dark matter density perturbations ($10^{12} M_{\odot}$) that define where galaxies will form
 - [2] Within DM perturbations, the first small clusters ($10^6 M_{\odot}$) collapse, begin forming the galactic **bulge** stars and globular clusters
 - [3] Collisions between small clusters disrupt star formation, collide into larger ($10^{10} M_{\odot}$) proto-galaxies that will become the **stellar halo**
 - [4] Residual torque from collisions and gravitational interactions induce angular momentum, causes matter to settle into the **thick disk**
 - [5] Cool gas settles into the **thin disk** at a smaller scale height

General and Special Relativity

In **Newtonian Relativity**, for an object moving at velocity \mathbf{v} in one frame, the apparent speed in a moving reference frame \mathbf{v}' is the **sum $\mathbf{v}+\mathbf{v}'$** . But this breaks down when velocities approach the **speed of light**

Special Relativity

- **Einstein's postulates of special relativity:**
 - **(1) principle of relativity:** the laws of physics apply in all **inertial** reference systems (reference frame is not accelerating)
 - **(2) the universal speed of light:** the speed of light in vacuum is the same for all inertial observers, regardless of the motion of the source
- **Lorentz factor:** $\gamma = \frac{1}{\sqrt{1-(v^2/c^2)}}$

- **Time dilation:** clocks in a moving reference frame measure a shorter time (run slow) than clocks in a rest frame: $\Delta t' = \sqrt{1 - v^2/c^2} \Delta t$
- **Length contraction:** length measured in a moving reference frame is shorter than length measured in rest frame: $\Delta x' = \frac{1}{\sqrt{1-v^2/c^2}} \Delta x$
- **Relativity of simultaneity:** Events that happen at the same time in one reference frame are not necessarily simultaneous in another
- **Lorentz Transformation** - convert from coordinates (x,y,z, and t) in one inertial frame S to coordinates (x',y',z', and t') in frame S' moving at speed **v** relative to frame S
 - $ct' = \gamma(ct - \beta x)$
 - $x' = \gamma(x - \beta ct)$
 - $y' = y, z' = z$
- **Energy and momentum:**
 - Einstein redefines **relativistic momentum** redefined using the **proper velocity** (velocity measured as change in x divided by **proper time** elapsed in the rest frame): $p = \frac{mv}{\sqrt{1-v^2/c^2}} = \gamma mv$
 - **Relativistic energy** is the 'time' component of relativistic momentum, multiplied by c: $E = \frac{mc^2}{\sqrt{1-v^2/c^2}} = \gamma mc^2$
 - **Rest energy** is when $\gamma = 1$, resulting in Einstein's famous equation: $E_{rest} = mc^2$
 - $E^2 = (pc)^2 + (mc^2)^2$
- **Synchrotron radiation**- requires charged particles to move near the speed of light; radiation is **relativistically beamed** in the direction of motion
- **Superluminal** motion- radio jets can appear to move faster than the speed of light due to **time dilation**

General relativity is a generalization of special relativity accounting for gravity in a **curved** 4-dimensional **spacetime** (x,y,z,t)

- (1) **Mass** tells **spacetime** how to curve; **spacetime** tells **mass** how to move
 - **Geodesic** - shortest path between two points within given geometry
 - **Photons/light** will always follow the geodesic, and therefore curves around massive objects

- (2) **Equivalence principle**: gravitational mass and inertial mass are the same; motion under gravity is identical to motion in an accelerating reference frame

- **Gravitational redshift** - light's wavelength increases (frequency decreases) when traveling away from a massive object, and decreases (frequency increases) when

travelling towards a massive object: $1 + z = \frac{v_{obs}}{v_0} = \frac{\lambda_0}{\lambda_{obs}} = \sqrt{1 - \frac{2GM}{r_0 c^2}}$

- **Gravitational time dilation** - time runs more slowly in curved spacetime:

$$\frac{\Delta t_0}{\Delta t_{obs}} = \sqrt{1 - \frac{2GM}{r_0 c^2}}$$

- **Black Holes** - mass is collapsed into a small radius such that the escape velocity equals the speed of light

- **Schwarzschild radius** $R_s = 2GM/c^2$ - minimum radius a given mass can occupy before becoming a black hole; also equals the radius of the **event horizon**

- **Gravitational lensing** - galaxies behind massive objects (black holes, other galaxies, DM overdensities) can be seen as light bends around the foreground object

- **Gravitational Waves** - ripples in spacetime due to the rapid motion of extremely dense objects like orbiting black holes and neutron stars in binaries

- We detect gravitational waves using **laser interferometry**; just like in radio interferometry, we cross-correlate the signals of two **in-phase/coherent** lasers travelling in perpendicular directions
- When a gravitational wave hits the laser, spacetime is compressed in one direction but unchanged in the other, causing the lasers to become **out-of-phase**, so we observe a **fringe-pattern**
- The **Laser Interferometer Gravitational-Wave Observatory (LIGO)**, **Virgo Interferometer**, and **Kamioka Gravitational Wave Detector (KAGRA)** use laser interferometry to detect gravitational waves

Hubble's Law and the Expanding Universe

Edwin Hubble observed that distant galaxies all had spectral lines that were **redshifted** (Doppler shifted to longer wavelengths) due to their motion, meaning that they were travelling **away from us**.

- **Hubble's Law: more distant galaxies moving away faster:** $v_r \approx cz \approx \frac{H_0}{c}D$ where $H_0 \approx 70 \text{ km/s/Mpc}$ is **Hubble's Constant**
 - But they can't all be moving away from Earth because the universe is **isotropic** - it has no preferred center or direction
 - Instead, all galaxies must be moving away from each other, and accelerating → **universe is expanding**, causing the spectral lines to experience **cosmological redshift**
- As we look at higher redshifts, we're looking **earlier** in time (because it takes longer for the light to reach us) when the universe was more compressed; the **Cosmic Microwave Background** is evidence that the universe is expanding from what once was a hot dense singularity
- Accelerating expansion implies the presence of **dark energy** which drives the expansion, though its nature is unknown and it cannot be detected