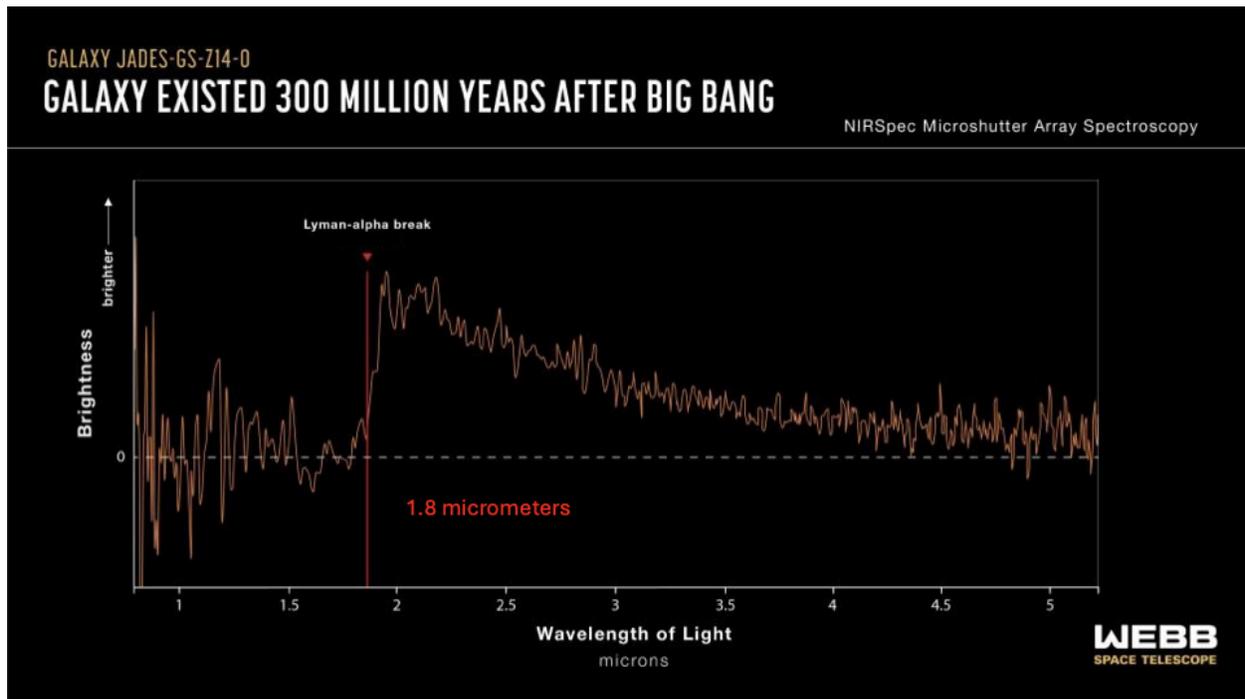


# Problem Set 4: Galaxies, Relativity, and Cosmology

Answer each question in 1 or 2 complete sentences, showing your work (math equations or illustrations) as needed.

1. The James Webb Space Telescope recently discovered the highest redshift galaxy ever! Below is the infrared (IR) spectrum:

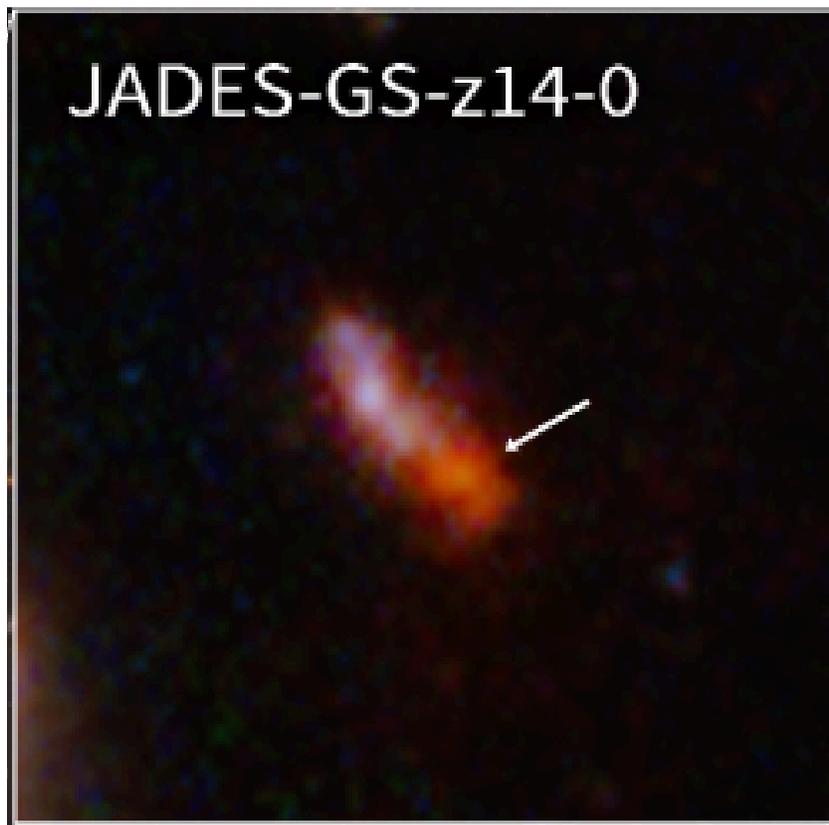


- a. The Lyman-alpha break is a cutoff seen in galaxy spectra, indicated by the red line above. Below this wavelength, most photons are absorbed by hydrogen; in the rest frame, the Lyman-alpha break is at  $\lambda_{emit} = 0.1216$  micrometers (microns). JWST detected it at  $\lambda_{obs} = 1.8$  microns. What is the redshift of the galaxy? (*Hint: redshift*

$$1 + z = \frac{\lambda_{obs}}{\lambda_{emit}}$$

b. Using a Hubble constant  $H_0 \approx 70 \text{ km/s/Mpc}$ , what is this galaxy's distance in Megaparsec? (*Hint: remember Hubble's law,  $cz = H_0 D$* )

c. Here is an infrared image of the galaxy; using Hubble's classification scheme, is this a dwarf, elliptical, or spiral galaxy? If it is a spiral, is it a barred or normal galaxy?



- d. What kinds of stars do you think this galaxy contains: Population I, Population II, or Population III? Why?

2. Believe it or not, the Christopher Nolan movie *Interstellar* is based on real and theoretical physics, including general relativity. Let's put it to the test! In one scene, the main characters visit a planet near a supermassive black hole and claim that "One hour on this planet is equal to 7 years on Earth," because of the effects of gravitational time dilation.

- a. In units of the Schwarzschild radius ( $r_s = \frac{2GM}{c^2}$ ) of the black hole, what is the distance to the black hole? (*Hint: for gravitational time dilation, the time that passes at an infinite distance from the black hole is*

$$\Delta t_{Earth} = \frac{\Delta t_{planet}}{\sqrt{1 - \frac{2GM}{rc^2}}}$$

- b. At one point, a character says they are trying to send data back to Earth, but it can't escape the black hole's gravitational pull. Why not?

c. *[Spoiler Alert!]* At the end of the movie, one of the astronauts returns home to find he now younger than his own daughter! Think about the twin “paradox” we discussed in class; does this make sense?

d. **Bonus:** The waves on the planet are enormous – so large that they initially think the waves are mountains! Tidal waves on Earth are caused by the gravitational pull of the moon on the ocean; similarly, these mountain sized tidal waves are caused by the gravitational pull of the black hole. The maximum height of a wave is  $h = \frac{3}{2} \frac{M_{BH} R_P^2}{M_P D^3}$ , where  $M_{BH}$  is the black hole mass,  $R_P$  is the planet’s radius,  $M_P$  is the planet’s mass, and  $D$  is the distance between the black hole and planet that you found in part a. If the wave they saw was the size of Mount Everest (8848 meters), what is the mass of the black hole?