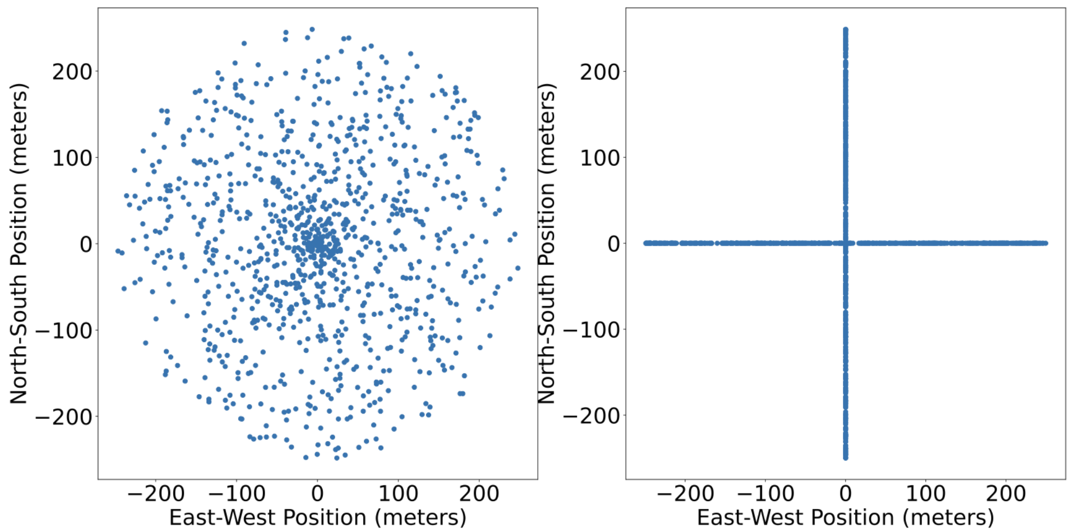
Problem Set 3: Radio Telescopes and Stellar Remnants

Answer each question in 1 or 2 complete sentences, showing your work (math equations or illustrations) as needed.

1. The Five-hundred metre Aperture Spherical Telescope (FAST) is the largest radio telescope in the world, with a 500 meter dish. It observes from 0.07 - 3 Gigahertz. But it cost $180 million dollars! Let's see if we can make an interferometer that's cheaper and just as sensitive.
   a. If we build our interferometer using 5-meter radio antennas, each individual antenna is 10000 times less sensitive than FAST. How many 5-meter telescopes will we need to match the sensitivity of FAST? *(Hint: the radiometer sensitivity for an interferometer is $\sigma \approx \frac{T_{sys}/G}{N\sqrt{\Delta\nu t_{int}}}$, where N is the number of antennas, Tsys is the system temperature, G is the gain, $\Delta\nu$ is the bandwidth, and t_{int} is the integration time.)*

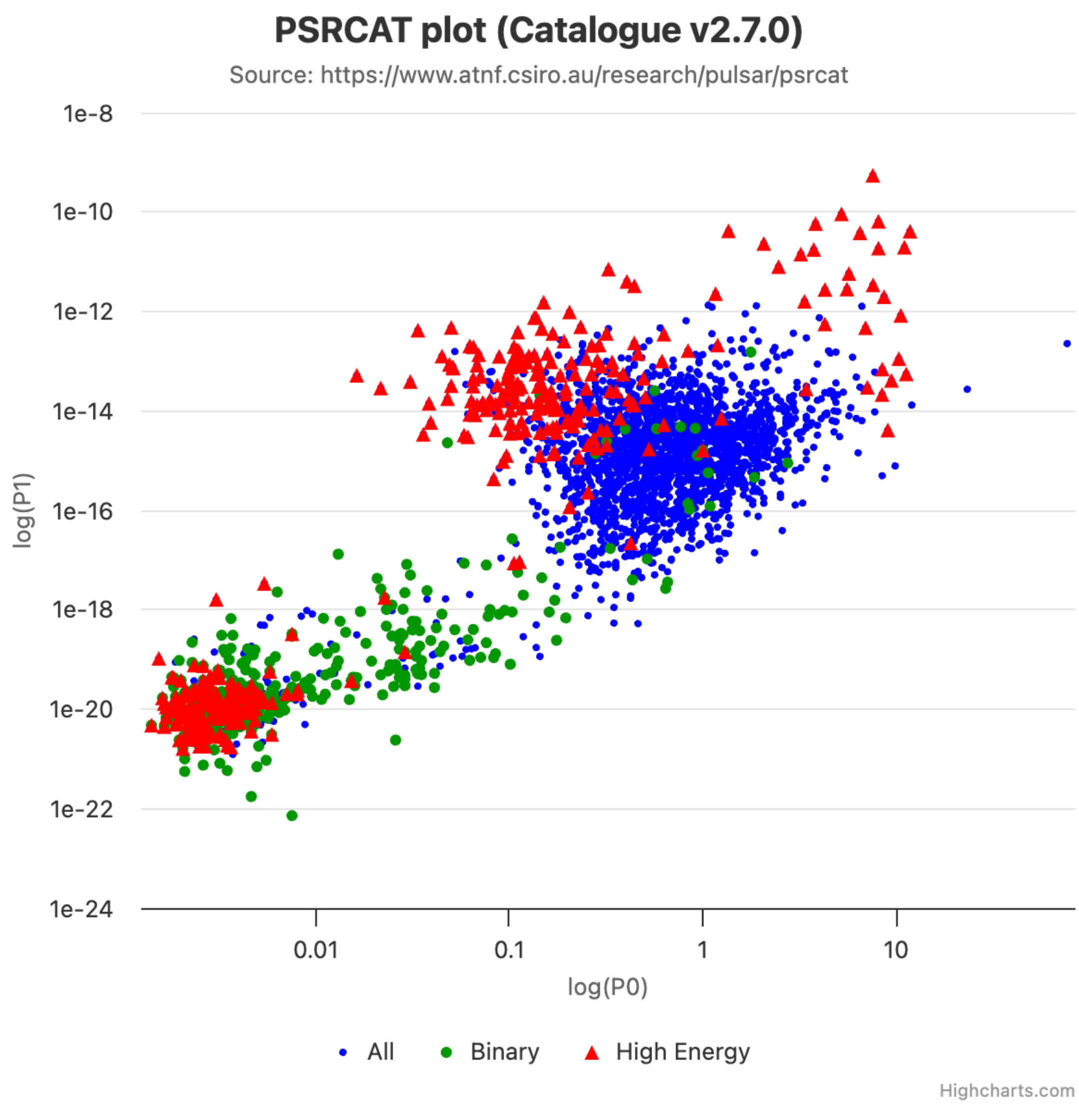
   b. If each antenna is $180k, we'll only have enough for 1000 antennas. How much should we increase the integration time to match FAST's sensitivity with 1000 antennas?

c. Below are two options for how to arrange our antennas; which should we use and why? (*Hint: think about the fringe patterns)*



d. ***Bonus:*** What is the full-width at half-max (FWHM) of FAST's primary beam? What about for each small telescope? What is the FWHM of the interferometer's synthesized beam if the antennas are randomly spread over a 500m circle (left figure)? Why is this an advantage for the interferometer? (*Hint: the full-width half max is* $\theta \approx 1.22\lambda/D$*, where you can use* $\lambda = c/1.4$ *GHz and D is the effective diameter)*

2. We've found a new pulsar, but it looks a bit…off. You were using the Green Bank telescope to search for pulsars and see radio pulses that repeat every 18.18 minutes! After monitoring the pulsar for a while, we find it has a spin-down rate of $\dot{P} = 6 \times 10^{-10} s/s$.
    a. What is different about this pulsar compared to other pulsars? For reference, here's a plot of the period (x-axis) and spin-down rate (y-axis) of pulsars from the ATNF Pulsar Catalog:



b. If we assume this is a pulsar which started with a rotation period close to 0, approximately how long has it been a pulsar? (*Hint: We can estimate a pulsar's age as* $\tau_C \approx P/\dot{P}$)

c. The brightest pulses have a radio luminosity $L = 4 \times 10^{31}$ erg/s. Most pulsars use their rotational kinetic energy to power their radio emission. How long could this 18.18 minute pulsar keep producing $L = 4 \times 10^{31}$ erg/s pulses? *(Hint: the rotational energy in ergs is* $E_{rot} \approx \frac{4\pi^2 \times 10^{45}}{2P^2}$*)*

d. Another theory is that this is not a pulsar, but a White Dwarf in binary orbit with an M-dwarf star, which usually has an intrinsic optical magnitude $M_V \approx 10$. If the distance is 1400 parsecs, and the extinction is around $A_V \approx 10$, what would the apparent optical magnitude be? (*Hint:* $m_V = M_V + A_V + 5log_{10}(d/1pc) - 5$*, and* $log_{10}(1400) = 3.14$*)*